Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 6, pp. 813–817, 2007

PROJECTILE DYNAMICS AT LOW BARREL PRESSURES

S. K. Chankaev and V. Ya. Yakovlev

UDC 531.571; 531.576

A mathematical model for a projectile shot at low pressures in the space behind the projectile space is developed. The pressure rise is limited because of the nonsimultaneity of propellant ignition and combustion and the discharge of the propellant combustion products through the gap between the projectile and the walls of the gun barrel. The kinetic characteristics of flame propagation over the propellant particles are determined. A comparison of calculation and experimental data is performed. The calculation results are used in designing 2A85 *self-propelled launchers and upgrading* 2A30 *self-propelled launchers.*

Key words: *pyrotechnics, interior ballistics, artillery projectile.*

This paper deals with modeling a projectile shot from a barrel at low pressures in the space behind the projectile. In the case of nonsimultaneous processes of propellant ignition and combustion and combustion gas discharge the gap between the projectile and the inner surface of the barrel, the pressure does not exceed several tens of atmospheres. This problem differs from the traditional problem of a projectile shot from artillery guns. In guns with a fairly high density of propellant charges, instantaneous propellant ignition is possible. At a rapidly increasing pressure (to several thousand atmospheres), propellant particles burn up in the space behind the projectile before the projectile leaves the barrel. Nevertheless, this problem has been analyzed, as a rule, using the traditional formulation of the problem of a projectile shot from an artillery gun with coefficients introduced to agree the results obtained with experimental data. To obtain physically valid results, we consider the formulation and solution of the problem of a projectile shot from a barrel taking into account flame propagation, propellant combustion at low pressures in the space behind the projectile, and propellant combustion gas discharge through the gap between the projectile and the inner wall of the barrel. The results of the present study can be useful in designing firework articles and launching devices for them.

1. Formulation of the Problem. In theoretical studies, the processes occurring in an artillery shot from guns are considered using traditional models. One of such models in a thermodynamic approximation is presented in [1]. According to this model, the kinematic characteristics of a projectile are found by solving the following system of equations, which includes the basic equation of pyrodynamics, the charge combustion law, the gas-formation law, and the equation of motion of the projectile, respectively:

$$PS(l_\psi + x) = fm\psi - \frac{k-1}{2}\,\varphi M V^2, \qquad \frac{de}{dt} = a_1 P,$$
$$\psi = \chi z + \lambda z^2, \qquad \varphi M\,\frac{dV}{dt} = PS. \tag{1}$$

Here x is the distance traveled by the projectile, V is the projectile velocity, P is the propellant gas pressure in the space behind the projectile, de/dt is the propellant burning rate, S is the cross-sectional area of the barrel, M is the projectile mass, m is the charge mass, $l_\psi = (1/S)(W - (1-\psi)m/\rho - \alpha m\psi)$ is the reduced free length of the space behind the projectile, ψ is the fraction of the propellant converted to gas, $m\psi$ is the mass of the gas, α is the covolume, ρ is the density of the propellant, W is the free volume under the projectile at the initial time, f is the

Piro-Ross Company, Sergiev Posad 141300, Moscow Region; piro-ross@tsinet.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 48, No. 6, pp. 44–49, November–December, 2007. Original article submitted December 20, 2006.

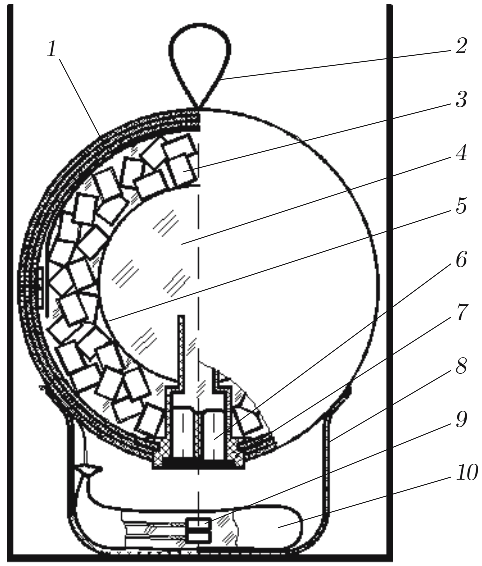

Fig. 1. Diagram of the charge: 1) projectile case; 2) loop; 3) pyroelements; 4) igniting-bursting charge; 5) case of the igniting-bursting charge; 6) delay element; 7) tube; 8) ejecting charge tube; 9) electric igniter; 10) ejecting charge.

propellant force, $k = c_p/c_v$, where c_p, and c_v are the specific heats at constants pressure and volume, respectively, a_1 is the propellant burning rate at $P = 1$ atm, $\varphi = 1.05$–1.10 is a coefficient that takes into account the effect of external forces, $z = e/e_0$ is the relative thickness of the propellant layer, where e and e_0 are the current and initial thicknesses of the propellant layer, and χ and λ are the coefficients characterizing the shape of the propellant particles.

The adequacy of the model is proved in [1] by comparing calculated projectile velocities at the exit from the barrel and the maximum propellant-gas pressure in the barrel with the corresponding values obtained experimentally. For the problem considered (at low pressures in the space behind the projectile), system (1) includes the additional coefficient ϕ which takes into account the nonsimultaneity of propellant ignition and the discharge of the combustion products through the gap between the projectile case and the inner wall of the barrel. In this case, the coefficient that takes into account the effect of external forces equals $\varphi = \phi(1.05$–$1.10)$. The value of ϕ for each case is determined by comparing experimental and calculated barrel pressures. In the middle of the time interval, the calculation and experiment results are in good agreement, whereas, at the initial times, the calculated values are much higher than the experimental data. Use of such estimates leads, in particular, to a considerable increase in the mass of launchers and does not allow one to determine the mass of the ejecting charge (EC) of the projectile required to reach the height of operation of the article. Such estimates do not take into account flame propagation over the EC propellant particles at fairly low pressures in the space behind the projectile and the discharge of the combustion products through the gap between the projectile and the inner wall of the barrel.

This paper is devoted to an analysis of the processes related to a projectile shot under the conditions formulated above.

2. Analysis of the Polydisperse Mode of Propellant Combustion. A barrel in the shape of a metal tube with welded bottom is considered. A projectile with an attached tube containing an EC in the form of a bag with a propellant is inserted in the barrel (Fig. 1). After operation of the igniting device, ignition and combustion of some portion of the propellant particles upon attainment of the fracture pressure in the tube, combustion occurs in the volume under the projectile. The pressure of the EC combustion products causes the projectile to move in the mortar barrel. After the exit from the barrel, the projectile moves mechanically.

The parameters of the problem are determined in the volume bounded by the inner surface of the barrel, the bottom surface of the projectile, and the surface perpendicular to the barrel axis, with the minimum gap between the projectile and the barrel (critical section) (Fig. 1). The propellant charge in the EC tube is initiated by one or two electric igniters. At the initial time, some part of the propellant charge is ignited. With time, the flame spreads over the entire propellant charge, involving new propellant particles in the combustion process. Gaseous and condensed propellant combustion products are formed. The mass fraction of the condensed combustion products is significant, $\varepsilon = 0.56$, which should be taken into account in the analysis. The condensed-phase particle size is assumed to be such that temperatures and velocities of the gas and condensed phases can be considered identical. The parameters of the model described here are determined by solving the following system, consisting of the equation of conservation of mass, the equation of conservation of energy, the equation of motion of the projectile and the equation of state, respectively:

$$\frac{dm_{\rm pr}}{dt} = \dot{\psi} - G, \qquad \frac{d\,(m_{\rm pr}u)}{dt} = -P\,\frac{dv}{dt} + \dot{\psi}u_0 - Gu,$$
$$M\,\frac{dV}{dt} = PS_u, \qquad P = (1-\varepsilon)\,\frac{m_{\rm pr}}{v}\,R\,\frac{u}{c_{vg}}. \tag{2}$$

Here $m_{\rm pr}$ is the mass of the propellant combustion products, ε is the fraction of the condensed phase in the propellant combustion products, $\dot{\psi}$ is the rate of formation of the propellant combustion products, G is the mass flow rate of the propellant combustion products through the critical section, u is the internal energy per unit mass of the combustion products, $u_0 = c_v T_b$, where T_b is the propellant combustion temperature, $v = W - m_*/\rho + Sx$ is the volume occupied by the propellant combustion products, m_* is the mass of the unburned propellant, S_u is the cross-sectional area of the projectile, R is the gas constant of the gaseous combustion products, and c_{vg} is the constant-volume specific heat of the gaseous combustion products.

To determine the quantity $\dot{\psi}$, we make a number of assumptions. It is assumed that the propellant particles have a spherical shape. The particle diameter $2r = 1$ mm. After the operation of the combustion initiators, some fraction N_0 of the propellant particles is ignited. The flame from the burning propellant spreads over the other particles. It is assumed that the burning and unignited propellant particles are uniformly distributed in the space behind the projectile and that the volume occupied by the particles is smaller than the volume in which the propellant combustion occurs. The flame propagation over the particles is represented as

$$\frac{dN}{dt} = \alpha_1 N\Big(\frac{P}{P_0}\Big)^{\alpha_2}, \tag{3}$$

i.e., the amount of ignited particles is proportional to the amount of burning particles in unit volume and to the pressure of the combustion products. Here α_i are constants determined from experiment. At the initial time, all particles have the same size. The mixture is monodisperse. During flame propagation, some particles are ignited, and the other cease to burn; therefore the combustion of the propellant particles should be considered using the model of a polydisperse medium. Equation (3) defines the law of formation of fractions of burning propellant particles. For different fractions of the propellant, the function δ_i is introduced: $\delta_i = 1$ if the fraction is burning, $\delta_i = 0$ is combustion has not yet begun or has already ended. The amount of ignited particles of the ith fractions is determined by integrating Eq. (3):

$$N_i = \alpha_1 \int\limits_{t_i}^{t_{i+1}} \Big(\sum_{j=0}^{i-1}(\delta_j N_j)\Big)\Big(\frac{P}{P_0}\Big)^{\alpha_2}\,dt. \tag{4}$$

Here $i = 1,\ldots,J_k$, $t_i = i\Delta$, $t_k = J_k\Delta$ is the time of the end of the process and Δ is the digitization parameter of the process. Expression (4) is valid if $\sum\limits_{j=0}^{i-1} N_j$ is smaller or equal to the initial amount of particles; otherwise, $N_i = 0$. Following [2], we assume that unitary propellants, which include gunpowder and explosives, contain not only fuel but also an oxidizer mixed with the fuel at the molecular level, i.e., they are a condensed solid homogeneous mixture of a fuel and an oxidizer. The linear burning rate of gunpowder and other types of unitary propellants depends on pressure. The corresponding empirical dependence is given by [3]

$$\frac{dr_i}{dt} = -b_1\Big(\frac{P}{P_0}\Big)^{b_2}, \tag{5}$$

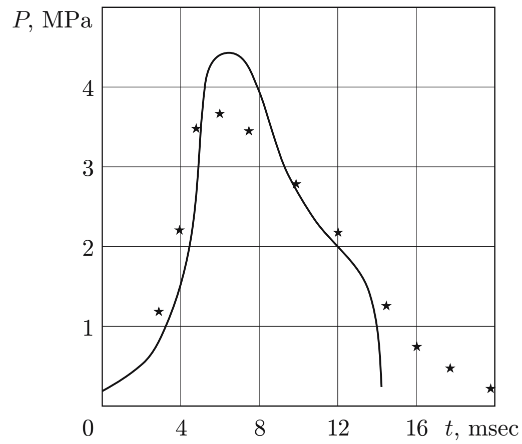

Fig. 2. Pressure at the barrel bottom: the curve refers to calculation results; the points are experimental data.

where b_1 and b_2 are empirical constants. For the propellant considered, $b_1 = 12.1$ mm/sec, $b_2 = 0.24$ at $P < 60$ MPa. During combustion of particles of the ith fractions, the value of r_i vanishes. At the same time, the function δ_i also vanishes. The function $\dot{\psi}$ can be expressed as

$$\dot{\psi} = 4\pi\rho \sum_i \delta_i N_i r_i^2 \left|\frac{dr_i}{dt}\right| \qquad (6)$$

(the summation is performed over all fractions).

The function G is determined by solving the problem of the discharge of combustion products from a supersonic nozzle [4]. A similar problem is considered in [5]. The gas flow rate through the critical section is determined from the relation

$$G = \frac{PF_*}{\sqrt{T}}\Big(\frac{2}{k+1}\Big)^{(k+1)/(2(k-1))}\Big(\frac{k}{R}\Big)^{1/2}. \qquad (7)$$

Here F_* is the critical-section area and T is the gas temperature in the space behind the projectile. For the mixture of the propellant combustion products, $k \approx 1.1$. From [4] it follows that supersonic flow of the propellant combustion products begins at $P > 1.63P_0$. In practice, this value is reached almost instantaneously.

System (2)–(7) allows one to solve the problem formulated.

3. Using the Method of Solution Developed. The examined formulation of the problem was used to study the behavior a firework article shot from a mortar barrel and to choose the article design parameters.

The high-altitude firework pyrotechnics produced in the Russian Federation are divided into two groups: 1) articles of calibers 195 and 310 mm operating at a height of 250 to 500 m and producing large figures from pyroelements of various standard sizes; 2) articles of calibers 60 and 105 mm operating at a height of 150 m, which, because of small dimensions, can be equipped with pyroelements of only one standard size. To make firework shows more spectacular, articles are required that act at a height of 150 to 250 m and produce voluminous figures. The importance of designing an article of caliber 125 mm is also due to the fact that mortars of this caliber can be installed on the existing 2A30 launchers without changing their base design. Before designing the article, we performed calculation studies. As the initial step, the constants appearing in system (4) were determined. The article of caliber 310 mm was studied the most extensively in experiments because it contains the maximum propellant charge and is of interest from the viewpoint of show safety. The constants N_0, α_1, and α_2 were determined by solving Eqs. (2)–(7) by the gradient method using the results of experiment with articles of caliber 310 mm. Figure 2 gives measured pressure of the combustion products at the barrel bottom for articles of this type. From the solution of the problem, it follows that, at the initial time, the electric igniters fire the propellant of mass ≈68 g. In this case, $\alpha_1 = 22$ and $\alpha_2 = 1.95$. An analysis of the calculations results for articles of caliber 125 mm shows that the kinetic constants remain unchanged whereas the mass of the propellant ignited at the initial time depends on the caliber of the article, in particular, for articles of caliber 125 mm, it is 35 g. The kinetics of propellant ignition and combustion is such that during the shot of an article of caliber 310 mm, ≈90 % of the ejecting charge burns up. The remaining burning propellant particles and combustion products are discharged from the mortar tube into the atmosphere, producing a burning smoky cloud behind the article.

Conclusions. A model was developed for a projectile shot from a barrel at low pressures in the space behind the projectile for nonsimultaneous propellant ignition and combustion and propellant combustion gas discharge through the gap between the projectile and the inner wall of the barrel. The calculation results show that the model is suitable for a wide class of projectiles used in pyrotechnics. In particular, the studies lead to the conclusion that for operation of an article of caliber 125 mm at a height of 150–250 m, it is sufficient to use an ejecting charge with a propellant mass of 72 g. Experimental results supported the validity of the choice of the propellant mass for the ejecting charge of the article of caliber 125 mm developed at the Piro-Ross company. This pyrotechnic article produces the most colorful and diverse firework pictures in the night sky with the use of 2A30 and 2A85 self-propelled launchers without significantly increasing the sizes of dangerous zones and without changing the base of the 2A30 devices. This makes it possible not only to design new firework systems but also to upgrade those available.

REFERENCES

1. M. E. Serebryakov, *Interior Ballistics* [in Russian], Oborongiz, Moscow (1949).
2. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Part 1, Hemisphere, New York (1991).
3. Ya. B. Zel'dovich, G. I. Barenblatt, V. B. Librovich, and G. M. Makhviladze, *The Mathematical Theory of Combustion and Explosions*, Plenum, New York (1985).
4. G. N. Abramovich, *Applied Gas Dynamics* [in Russian], Nauka, Moscow (1976).
5. L. Weinman, "Choked flow, a frequently misunderstood term," *J. Pyrotech.*, No. 21, 31–35 (2005).